

6-man Chess and Zugzwangs

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Abstract. With 6-man Chess essentially solved, the available 6-man Endgame Tables (EGTs) have been scanned for zugzwang positions where, unusually, having the move is a disadvantage. Review statistics together with some highlights and positions are provided here: the complete information is available on the ICGA website. An outcome of the review is the observation that the definition of *zugzwang* should be revisited, if only because the presence of *en passant capture* moves gives rise to three new, asymmetric types of zugzwang.

Keywords: chess, endgame tables, maximal, Nalimov, zugzwang

1 Introduction

Six-man Chess is essentially solved and the Nalimov [1] Endgame Tables (EGTs) to DTM, Depth to Mate, have been widely promulgated for some time [2]¹. The corpus of perfect information is a challenge to datamine for both helpful guidelines and for the pathological positions – the deep, exceptional, bizarre and amusing.

A *zugzwang* position is defined here as one where the side to move would prefer that it were the other side's turn to move. They are remarkable in themselves and the 'zug', much sought after by composers, is a running theme in the Study community [3-9]. The zug is also the counterexample to the assumption of the Null Move Heuristic that having the move is an advantage. When 5-man chess was solved, the two sets of respectively DTM and DTC² [10] EGTs were searched [11] for zugs³. A later review of Thompson's 6-man pawnless DTC EGTs also included a zug search [12].

The zugzwang search reported here is almost entirely of the highly available [2, 13] Nalimov DTM EGTs. The search was carried out via the web using Bourzutschky's GTBGEN and the first author's EGTs and JAVA code. Unpublished FEG DTM EGTs [14] for 5-1(p) chess and DTC EGTs for 6-man chess [15-16] exist and Bourzutschky has kindly supplied [16] a zug review from the latter of 5-1p chess.

Section 2 considers *zugzwang* definitions and identifies three new types of zug; section 3 covers logistics. Section 4 is a summary of the main findings. Tables 1-5 list various illustrative statistics and positions; the full details are available via the ICGA website [17] which will host the evolving story of the zugzwang.

¹ *Essentially* because positions with castling rights are not yet included in EGTs.

² DTC ≡ Depth to Conversion, i.e. to force-change and/or Mate.

³ Incidentally providing a partial cross-check of agreement between the two sets of EGTs.

2 Definitions of *The Zugzwang*

Zugzwang is defined [18] as ‘pressure to take action’ and a *zugzwang position* is defined to be one where this pressure is unwelcome – where the first player would rather ‘pass the position across’.⁴ However, in [19] a *zugzwang position* is defined as ‘a position in which whoever has the move would obtain a worse result than if it were the opponent’s turn to play’. Note that this brings in the 2nd player’s perspective but focuses only on the outcome without considering its achievement or likelihood. Other authorities refer to zugzwangs as *reciprocal zugzwangs* or *mutual zugzwangs*. The words *whoever*, *reciprocal* and *mutual* suggest a symmetry, perhaps assuming incorrectly that the 2nd player can always pass back the 1st position to the 1st player.

Consider the *en passant zone EPZ* of Chess, i.e., those positions where there is an en passant capture option. Let $p1 \in EPZ$: what now are positions $p2$ and $p3$? The proposal here⁵ is to clarify this situation by formalising the notions of *passing over* or *losing the move* as one of *playing a null move* or *nulling*. Now the rules of chess define $p2$ and $p3$: the e.p.-capture option in $p1$ disappears if not played immediately, $p2$ does not feature any e.p.-capture option as the 1st player has not moved a Pawn two squares, and $p3$ is $p1$ without the e.p.-capture option. Positions in EPZ are only 1.4% of those in their endgame and zugs in EPZ comprise only 0.22% of the total.

Let the *Level A zugzwang*, our focus here, be described in these terms:

- a) positions are valued from 1st player’s perspective: loss (0), draw (1), win (2),
- b) if there is force-symmetry and/or no *e.p.*, 1st player is assumed to be White
1st player is Black in eleven of Table 1’s positions,
- c) 1st player in position $p1$ (value $v1$) nulls to $p2$ (value $v2$) iff $v2 > v1$,⁶
- d) 2nd player may *or may not*, c.f. Table 1’s Z08-9, null to $p3$, value $v3 \leq v2$,
- e) if $p3 \equiv p1$, $v3 \equiv v1$. If $p3 \neq p1$, $p3$ is ‘stalemate’ or $v3 \leq v1$.

Fig. 1 is formatted so that the 1st player nulls ‘to the right’ and the 2nd player nulls to the left to increase value to themselves. Clearly, in addition to the three familiar zug types 1-3,⁷ we now have, exclusively in the EPZ, just three more types 4-6:⁸

- 1) type 1 \equiv ‘121’ \equiv ‘draw-win-draw’, q.v. Z01 in Table 1
- 2) type 2 \equiv ‘010’ \equiv ‘loss-draw-loss’, q.v. Z02: t1-t3 are *no net gain* for player 1
- 3) type 3 \equiv ‘020’ \equiv ‘loss-win-loss’, q.v. Z03: the *full-point* zug
- 4) type 4 \equiv ‘120’ \equiv ‘draw-win-loss’, q.v. Z05-6: t4 is a *net loss* for player 1
- 5) type 5 \equiv ‘021’ \equiv ‘loss-win-draw’, q.v. Z07: t5 is a *net gain* for player 1
- 6) type 6 \equiv ‘01(1)’ \equiv ‘loss-draw(-draw)’, q.v. Z08-9: t6 is a *net gain* for player 1⁹

Clearly, type 4-6 zugs are asymmetric. Considered only in terms of the first two positions, type 4 becomes type 1, type 5 becomes type 3 and type 6 becomes type 2.

⁴ Not strictly possible, as in ‘passing the position across’, the side to move changes.

⁵ Our project log notes that Bourzutschky proposed the *Null Move* concept on 2005-05-31.

⁶ *Value* is calculated as normally, assuming that the option of a null move is not available.

⁷ Types are distinguished by the sequence $v_1-v_2-v_3$ rather than just by the sequence v_1-v_2 .

⁸ Unless position $p3$ is *stalemate*, its value $v3 \leq v1$ as the 1st player has no less moves in $p1$.

⁹ The 2nd player may even prefer to play on rather than stalemate their opponent.

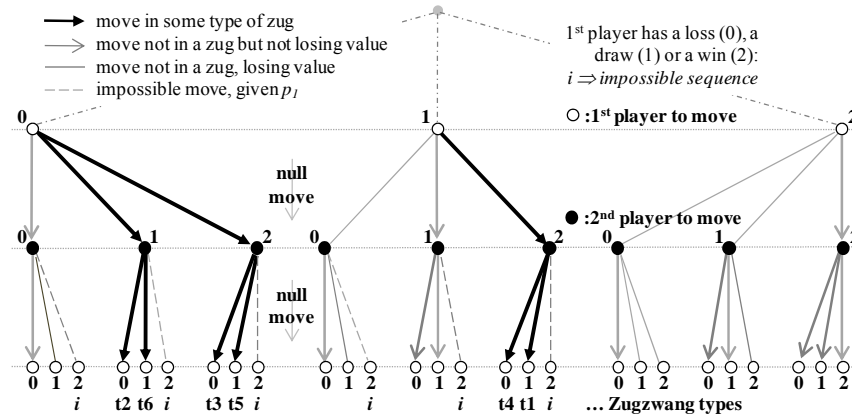


Fig. 1. The six types of zugzwang: the familiar t1-t3 and the ‘EPZ’ t4-t6.

Table 1. Some examples of zugzwangs of different types.^{10,11}

| id | Endgame | Position | Zug | | value and DTx | | | |
|-----|------------|---|------|-----|---------------|--------|-----|------|
| | | | Type | p1 | p2 | p3 | x | Flag |
| Z01 | KPK | 1k6/1P6/2K5/8/8/8/8 w | A1 | 121 | = | +2 | = | Z |
| Z02 | KBKP | 8/8/8/8/8/1pK5/kB6 w | A2 | 010 | -1 | == | -1 | Z m |
| Z03 | KPKP | 8/1pK5/kP6/8/8/8/8 w | A3 | 020 | -1 | +1 | -1 | Z m |
| Z04 | KPPKP | 8/8/8/3k4/2pP4/2K5/1P6/8 b - d3 | A2 | 010 | -25 | = | -15 | M |
| Z05 | KBPKPP | 8/8/8/1p6/1Pp5/8/4K3/2kB4 b - b3 | A4 | 120 | = | +21 | -20 | M u |
| Z06 | KPPKPP | 8/1p6/1k6/pP6/K7/P7/8/8 w - a6 | A4 | 120 | = | +21 | -30 | M m |
| Z07 | KP(5)KP(4) | 8/8/8/2p5/1pP1p3/kP2P3/Pp1P4/1K6 b - c3 | A5 | 021 | -0 | +1 | == | Z |
| Z08 | KRPKPP | 8/8/8/8/pP6/p7/k1K5/1R6 b - b3 | A6 | 011 | -0 | = | == | Z u |
| Z09 | KPPPKP | 8/8/8/8/1pP5/kP6/P7/K7 b - c3 | A6 | 011 | -0 | == | == | Z u |
| Z10 | KRNKNN | 8/8/8/8/2n5/1n6/R1N5/3K1k2 w | A2 | 010 | -1 | = | -1 | M |
| Z11 | KRBNKNN | 8/8/8/8/2n5/1n6/R1N5/1B1K1k2 w | A3 | 020 | -1 | +38(+) | -1 | M |
| Z12 | KBPKNP | 2n5/8/8/2B5/1Pp5/k1K5/8/8 b - b3 | A2 | 010 | -18 | = | -18 | M i |
| Z13 | KQBKNP | 8/1K6/8/2k5/Q1Bn4/8/2p5/8 w | A1 | 121 | = | +21 | = | M U |
| Z14 | KQNKBN | 8/8/8/8/8/nkb5/1N6/Q1K5 w | A1 | 121 | = | +35 | = | M U |
| Z15 | KQNKNN | 8/8/8/n1Q5/2n5/8/2kn4/K7 w | A1 | 121 | = | +35 | = | M U |
| Z16 | KQRKBB | 8/8/6b1/2R5/3K4/4Q3/5b2/5k2 w | A1 | 121 | = | +26 | = | M U |
| Z17 | KQRKRB | 2r5/R1Q5/b7/8/8/2K5/8/1k6 w | A1 | 121 | = | +17 | = | M U |
| Z18 | KQBPKB | 1k6/1P6/K7/1Q6/B1b5/8/8/8 w | A1 | 121 | = | +9 | = | M U |
| Z19 | KQBPKP | 8/8/8/8/8/1p6/1PQ5/kBK5 w | A1 | 121 | = | +11 | = | M U |
| Z20 | KRRRKQ | 8/8/8/8/8/4q1k1/2R5/1R1K3R w | A1 | 121 | = | +22 | = | M U |
| Z21 | KBPKNP | 3n4/8/8/5pP1/8/8/8/1kBK4 w - f6 | A1 | 121 | = | 32 | = | M |
| Z22 | KBPKNP | 8/8/3B2n1/K7/1pP5/k7/8/8 b - c3 | A2 | 010 | -32 | = | -32 | M |
| Z23 | KNPKPP | 8/8/8/p7/Pp6/3N4/3K4/k7 b - a3 | A2 | 010 | -25 | = | -25 | M |
| Z24 | KRPKPP | 8/1K6/1p6/2k5/1pP5/8/8/2R5 b - c3 | A2 | 010 | -21 | = | -21 | M |
| Z25 | KBPPKP | 1BK5/8/k7/Pp6/8/P7/8/8 w - b6 | A1 | 121 | = | +17 | = | M |
| Z26 | KNPPKP | 8/8/K7/PNk5/Pp6/8/8/8 b - a3 | A2 | 010 | -23 | = | -23 | M |
| Z27 | KPPPKP | 8/8/1k6/1P6/KPp5/8/P7/8 b - b3 | A2 | 010 | -17 | = | -17 | M |

¹⁰ Values from 1st player’s perspective: + win, = draw, == stalemate, - loss.

¹¹ Flags: *m* maxDTx, *i* inaccessible, *s* symmetric, *u* unique of type, *U* unique in endgame.

3 Enumeration: Endgames and Zugzwang Occurrences

These notes explain the lexical ordering of men and of endgames, and the principles used for counting the occurrences of zugzwangs.

The men are listed in the *strength order* K-Q-R-B-N-P. White has at least as many men as Black. In *m-m* endgames, White's *lead men* are at least as strong as Black's.

No attempt is made to eliminate unreachable positions in EGT statistics: this is usual as there is no general algorithm. With this limitation, the count is of FEN-distinct and functionally unique zugzwangs. Thus, no zugzwang can be physically transformed into any other. The following subtleties should be noted:

- a) For force-symmetric zugs $z \notin \text{EPZ}$, type 1 and type 2 zugs are equivalent: the count of type 2 zugs is shown in brackets,
- b) For force-symmetric zugs, type 3 zugs usually appears in two physical forms:
 - the two physical versions were identified¹² and counted as one here,
- c) When both Kings are on a1-h8 or a8-h1 in pawnless endgames:
 - Nalimov has both physical versions of the position if there are two;
 - the two physical versions were identified and counted as one here,
- d) When e.p.-capture has been enabled but is actually illegal, q.v. Z12:
 - the position is counted here as different from that without the *e.p.*,
 - 1st player would have to realise that the e.p.-option is illusory,
 - FIDE's recently reworded Article 9.2 now seeks to ignore the *e.p.* [20-22],
- e) An example of an unreachable zug:
 - position Z12 also implies the prior 1. b2-b4, impossible on two grounds:
 - before 1. b2-b4, the side not to move, Black, is in (double) check,
 - check from a Pawn on its home square is itself impossible,
 - therefore, the position prior to Z12 is also unreachable.

We note one small, historical and now resolved hiatus with respect to these results. The identification of zugs of types 4-6 was a serendipitous accident¹³ and was initially regarded as a bug by GTBGEN's author Marc Bourzutschky.

When the 'last 16' 3-3p endgames to KPPKPP were published, both by MB converting his FEG EGTs to Nalimov's format¹⁴ [23] and by Nalimov returning a disc to Hernandez [24], MB discovered that although he had anticipated a 2-byte format for the KQPK(B/R)P EGTs in GTBGEN, Nalimov had in fact discovered that the 1-byte format would suffice [25]. MB realigned with Nalimov and removed the ability to detect type 4-6 zugs before a GTBGEN was provided that could address these two EGTs. MB now advises [16] that no type 4-6 KQPKBP or KQPKRP zugs exist.

The full results have been published as a set of downloadable files [17]. Table 2 provides summary statistics for the six blocks of 6-man zugs, 3-3(p), 4-2(p) and 5-1(p). There are 293 5-1p zugs, 232 featuring a touring Knight on the wrong foot.

Table 3 details the occurrences of e.p. zugs including the new types of zug. Table 4 lists in lexical order all endgames with type 3 zugs, together with an example of their deepest type 3 zug. Table 5 brings together various kinds of position: type 2 e.p. zugs

¹² The identification of symmetries and equivalences was done by the first author's code.

¹³ The serendipitous accident has its place of honour in the history of discovery.

¹⁴ After reverse-engineering the unpublished format of the FEG EGT data format.

with p3's depth less than p1's, a type 3 zug N01 with e.p., zugs destroyed by the addition of an e.p.-capture or castling option, zugs unaffected or created by the addition of castling rights, 5-1p and 7-man zugs. N01-8 were discovered by Elkies [26] and B01-6 by Bourzutschky [15-16].

Table 2. Some summary statistics per 6-man endgame group.^{15,16}

| Item \ Group | 3-3 | 4-2 | 5-1 | 3-3p | 4-2p | 5-1p | Total |
|------------------------|----------|--------|-----|----------|---------|------|------------|
| 01 Endgames | 55 | 80 | 35 | 65 | 95 | 35 | 365 |
| 02 <i>No zugs</i> | 12 | 50 | 35 | 12 | 29 | 28 | 138 |
| 03 No t1 zugs | 12 | 50 | 35 | 12 | 30 | 28 | 139 |
| 04 No t2 zugs | 31 (+ 9) | 71 | — | 22 (+ 3) | 51 | — | 175 (+ 12) |
| 05 No t3 zugs | 55 | 80 | — | 50 | 67 | — | 252 |
| 06 <i>A unique zug</i> | 4 | 1 | 0 | 1 | 2 | 0 | 8 |
| 07 One t1 zug | 4 | 1 | 0 | 1 | 4 | 0 | 10 |
| 08 One t2 zug | 1 | 2 | — | 1 | 6 | — | 10 |
| 09 One t3 zug | 0 | 0 | — | 6 | 3 | — | 9 |
| 10 t1-t6 zugs | 27,597 | 20,017 | 0 | 379,218 | 478,682 | 293 | 905,807 |
| 11 t1 zugs | 27,470 | 8,434 | 0 | 361,712 | 373,479 | 293 | 771,388 |
| 12 t2 zugs | 127 | 11,583 | — | 15,543 | 105,069 | — | 132,322 |
| 13 t3 zugs | 0 | 0 | — | 1,568 | 133 | — | 1,701 |
| 14 t4 zugs | — | — | — | 394 | 0 | — | 394 |
| 15 t5 zugs | — | — | — | 0 | 0 | — | 0 |
| 16 t6 zugs | — | — | — | 1 | 1 | — | 2 |

Table 3. Statistics for the fourteen endgames with *e.p.-zugs*.

| Endgame | e.p.-zugs | | Type 1 | | Type 2 | | Type 3 | | Type 4 | | Type 5 | | Type 6 | |
|---------------|--------------|-----------|--------------|-----------|-----------|-----------|----------|----------|------------|----------|----------|----------|----------|----------|
| | wtm | btm | wtm | btm | wtm | btm | wtm | btm | wtm | btm | wtm | btm | wtm | btm |
| KPPKP | 35 | 20 | 35 | 0 | 0 | 20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| KBPKBP | 10 | — | 10 | — | 0 | — | 0 | — | 0 | — | 0 | — | 0 | — |
| KBPKNP | 130 | 5 | 130 | 2 | 0 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| KBPKPP | 18 | 18 | 18 | 4 | 0 | 13 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| KNPKNP | 156 | — | 156 | — | 0 | — | 0 | — | 0 | — | 0 | — | 0 | — |
| KNPKPP | 250 | 19 | 250 | 17 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| KPPKPP | 1,301 | — | 869 | — | 39 | — | 0 | — | 393 | — | 0 | — | 0 | — |
| KQPKQP | 75 | — | 72 | — | 3 | — | 0 | — | 0 | — | 0 | — | 0 | — |
| KRPKBP | 20 | 7 | 20 | 0 | 0 | 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| KRPKNP | 27 | 14 | 27 | 0 | 0 | 14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| KRPKPP | 0 | 2 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| KBPPKP | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| KNPPKP | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| KPPPKP | 8 | 2 | 8 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| Totals | 2,031 | 88 | 1,596 | 23 | 42 | 62 | 0 | 0 | 393 | 1 | 0 | 0 | 0 | 2 |

¹⁵ (+n) indicates that there are n endgames whose type 2 zugs merely mirror their type 1 zugs.

¹⁶ The eight zugzwangs which are unique across their endgames are Z12-Z19 in Table 1.

Table 4. The 43 endgames with *full-point* type 3 zugs: maxDTM examples.^{17,18,19}

| id | Endgame | Position | val / DTM | | | Total | Flag |
|-----|---------|-------------------------------|-----------|------|------|-------|------|
| | | | p1 | p2 | p3 | | |
| F01 | KBPKNP | k7/Bp4n1/1K1P4/8/8/8/8 w | -25 | +16 | -25 | 41 | |
| F02 | KBPKPP | 8/8/8/8/3K1k1p/3P2p1/6B1/8 w | -57 | +28 | -57 | 85 | |
| F03 | KNPKNP | 8/8/8/8/8/K1k5/P1p5/n1N5 w | -13 | +24 | -13 | 37 | u |
| F04 | KNPKPP | 8/8/8/8/4kp1p/3N4/2KP4/8 w | -71 | +26 | -71 | 97 | |
| F05 | KPPKPP | 8/8/8/8/p4p2/k1P5/2K1P3/8 w | -17 | +103 | -17 | 120 | |
| F06 | KQNKQP | 8/8/8/3N4/8/k2p4/3q4/KQ6 w | -14 | +6 | -14 | 20 | u |
| F07 | KQNKRP | QN6/Kpk5/1r6/8/8/8/8 w | -8 | +32 | -8 | 40 | u |
| F08 | KQPKQP | 8/8/8/1Pq5/8/1K1Q4/5p2/2k5 w | -24 | +95 | -24 | 119 | |
| F09 | KQPKRB | 2K1k3/2P5/8/8/8/1r2b3/4Q3 w | -20 | +15 | -20 | 35 | u |
| F10 | KQPKRP | 8/8/8/1Q6/8/1pP5/2k2r2/K7 w | -12 | +42 | -12 | 54 | |
| F11 | KRNKNP | 8/8/8/8/p7/N2k4/RK1n4 w | -1 | +28 | -1 | 29 | u |
| F12 | KRNKPP | 8/8/8/2N5/8/7p/k5pR/2K5 w | -33 | +17 | -33 | 50 | |
| F13 | KRPKBP | 8/8/8/8/2p5/2Pk4/1KR1b3 w | -15 | +30 | -15 | 45 | |
| F14 | KRPKNP | 8/8/8/8/3n4/k7/p1P5/K1R5 w | -1 | +33 | -1 | 34 | u |
| F15 | KRPKPP | 8/8/6R1/k1P5/2K5/7p/6p1/8 w | -71 | +9 | -71 | 80 | |
| F16 | KBBPKQ | 8/8/8/8/1K6/BBP5/8/qk6 w | -102 | +13 | -102 | 115 | |
| F17 | KBNPKB | K7/P1k5/8/8/8/6N1/5B1b w | -2 | +14 | -2 | 16 | |
| F18 | KBNPKN | BK1n4/NP1k4/8/8/8/8/8 w | -1 | +32 | -1 | 33 | |
| F19 | KBPPKB | K7/P1k5/8/8/8/6P1/5B1b w | -2 | +14 | -2 | 16 | |
| F20 | KBPPKP | 8/8/8/8/3k4/1K1p4/1P3P2/B7 w | -18 | +16 | -18 | 34 | |
| F21 | KBPPKQ | 3K1kq1/8/4PB2/3P4/8/8/8/8 w | -34 | +40 | -34 | 74 | |
| F22 | KBPPKR | 8/8/8/8/8/2k5/P1P5/rBK5 w | -18 | +22 | -18 | 40 | |
| F23 | KNNPKN | K1k5/P2N4/4N3/3n4/8/8/8/8 w | -1 | +19 | -1 | 20 | |
| F24 | KNPPKN | K7/P1kN4/8/3P4/n7/8/8/8 w | -2 | +20 | -2 | 22 | |
| F25 | KNPPKP | 8/8/8/K7/P1k5/1p6/3P4/4N3 w | -20 | +23 | -20 | 43 | |
| F26 | KNPPKQ | 1K1k2q1/8/2P5/3N4/8/2P5/8/8 w | -20 | +45 | -20 | 65 | |
| F27 | KNPPKR | N1k5/2P5/rPK5/8/8/8/8/8 w | -22 | +12 | -22 | 34 | |
| F28 | KPPPKN | n7/P1k5/K7/PP6/8/8/8/8 w | -2 | +13 | -2 | 15 | |
| F29 | KPPPKP | 8/8/8/5k2/3K1p2/3P3P/3P4/8 w | -20 | +19 | -20 | 39 | |
| F30 | KPPPKQ | k7/q1PK4/P7/8/8/2P5/8/8 w | -15 | +19 | -15 | 34 | |
| F31 | KPPPKR | 1K6/1P1k4/1r6/1P6/2P5/8/8/8 w | -21 | +36 | -21 | 57 | |
| F32 | KQNPKN | QN6/KP6/8/nk6/8/8/8/8 w | -1 | +8 | -1 | 9 | |
| F33 | KQNPKQ | 8/8/8/5N2/1q6/8/Q2P4/K1k5 w | -3 | +35 | -3 | 38 | |
| F34 | KQPPKQ | 8/8/8/1q6/5P2/P7/Q7/K1k5 w | -4 | +33 | -4 | 37 | |
| F35 | KRBPKB | K7/P1k5/8/8/8/6R1/5B1b w | -2 | +14 | -2 | 16 | |
| F36 | KRBPKN | RK6/B3n3/1Pk5/8/8/8/8/8 w | -2 | +14 | -2 | 16 | u |
| F37 | KRBPKP | 8/8/8/8/1k6/pP6/BRK5 w | -11 | +21 | -11 | 32 | |
| F38 | KRBPKQ | 1qk5/8/RBP5/8/8/8/1K6 w | -41 | +13 | -41 | 54 | u |
| F39 | KRNPKN | 8/8/8/8/n7/P7/K1k5/RN6 w | -1 | +24 | -1 | 25 | |
| F40 | KRNPKQ | 1K3N2/3R1P2/1kq5/8/8/8/8/8 w | -11 | +16 | -11 | 27 | |
| F41 | KRPPKN | K7/P1k5/R1P5/8/2n5/8/8/8 w | -1 | +20 | -1 | 21 | |
| F42 | KRPPKQ | 3R4/q7/k1P5/P7/K7/8/8/8 w | -36 | +12 | -36 | 48 | u |
| F43 | KRPPKR | 8/8/8/8/rPK5/1RP5/2k5 w | -26 | +29 | -26 | 55 | |

¹⁷ The depth of a type 3 or type 5 zug is defined as the sum of the depths of *p1* and *p2*.

¹⁸ KPPKPP zug F05 is the deepest type 3 zug with *dtm* = 120.

¹⁹ KBPPKQ zug F21 has the maximal depth 'shallower side' loss in this table.

Table 5. More didactic positions including 5-1p zugs and e.p. and/or castling effects.²⁰

| id | Endgame | Position | DTx | | | | Flag |
|-----|-----------------|--|------------------------|-----|-----|-----|-------|
| | | | Type | p1 | p2 | p3 | |
| P01 | KQNRRR | 1KQNk2r/7r/8/8/8/8/8 w - - | A1 | = | +54 | = | Z c |
| P02 | KRRKRB | r3kb2/1RK4R/8/8/8/8/8 w - - | A1 | = | +7 | = | Z c |
| P03 | KPPKPP | 8/6p1/4k1P1/4Pp2/3K4/8/8/8 w - f6 | A2 | -24 | = | -19 | M e |
| P04 | KNNKP | 7k/8/5NK1/7p/8/8/N7/8 b - - | B | -0 | -1 | -0 | Z |
| P05 | KPPKPP | 8/8/p2k4/6p1/3K2P1/P7/8/8 b - - | C | = | = | = | Z |
| P06 | KBPPPK | 5kBK/5P1P/7P/8/8/8/8 w - - | A1 | = | +2 | = | Z 5 |
| P07 | KBPPPK | 7K/5kBP/5P1P/8/8/8/8 w - - | A1 | = | +2 | = | Z 5 |
| P08 | KBPPPK | 5kBK/5P1P/8/3P4/8/8/8/8 w - - | A1 | = | +2 | = | Z 5 |
| P09 | KBPPPK | 5k1K/5P1B/5P2/5P2/8/8/8/8 w - - | A1 | = | +2 | = | Z 5 |
| P10 | KBPPPK | 8/B1k5/K7/P7/P7/P7/8/8 w - - | A1 | = | +3 | = | Z 5 |
| P11 | KBPPPK | 1k6/8/KP6/BP6/1P6/8/8/8 w - - | A1 | = | +1 | = | Z 5 |
| P12 | KNPPPK | 7K/5k1P/4N2P/7P/8/8/8/8 w - - | A1 | = | +2 | = | Z 5 |
| P13 | KPPPKK | 5k2/5P2/4K3/7P/7P/8/8 w - - | A1 | = | +2 | = | Z 5 |
| P14 | KQNPPK | 4k1KQ/5NPP/8/8/8/8/8 w - - | A1 | = | +2 | = | Z 5 |
| P15 | KRPPPK | 1k6/1P6/K7/RP6/P7/8/8/8 w - - | A1 | = | +2 | = | Z 5 |
| N01 | KPPPKPPP | 8/1p6/8/pP4pK/5kP1/P7/8/8 w - a6 | A3 | -1 | +1 | -1 | Z e |
| N02 | KPPPK | 3K4/8/3k4/8/3Pp3/4P3/8/8 b - - | A2 | -4 | = | -4 | Z e |
| N03 | KPPKPP | 8/8/8/5pK1/4kPp1/8/7P/8 w - - | A3 | -1 | +1 | -1 | Z e |
| N04 | KQP(6)KRRBP(3) | Q1K1k2r/PPP1p2p/b1r1P2P/2p5/2P5/8/8/8 w k - | A2 | -1 | = | -1 | Z c |
| N05 | KQP(6)KRRBP(5) | Q1K1k2r/PPP1p2p/bprpP2P/2p5/2P5/8/8/8 w k - | A3 | -1 | +2 | -1 | Z c |
| N06 | KQP(8)KRRBP(7) | Q1K1k2r/PPP1p2p/bprpP1pP/2p5/2P2pP1/8/5P2/8 b k g3 | A3 | -2 | +2 | -2 | Z c e |
| N07 | KRBNP(3)KRBP | r3k1KR/3p2PB/3P2N1/3P3b/8/8/8/8 w q - | A3 | -1 | ? | -1 | Z c |
| N08 | KRBNP(4)KRBP(3) | r3k1KR/3p2PB/2p2N1/7b/1pP5/8/1P6/8 b q c3 | A3 | -? | +1 | -? | Z c e |
| B01 | KNNKNN | 7k/8/4N3/4NN2/n2K4/8/8/3n4 w - - | A1 | = | +17 | = | Z s |
| B02 | KRBBKQB | 8/8/8/8/2b2q2/B7/1R3B2/2k1K3 w - - | A3 | -96 | +2 | -96 | Z u |
| B03 | KRBBKQN | 8/5B1q/6R1/3n4/8/8/2KB4/k7 w - - | A3 | -6 | +2 | -6 | Z |
| B04 | KRRRKR | 8/8/8/8/3Rr3/kr6/2KRR3 w - - | A1 | = | +2 | = | Z U |
| B05 | KBBBBKQ | 6B1/1B4qB/5k2/8/3K4/8/6B1/8 w - - | A2 | -35 | = | -35 | Z s |
| B06 | KBBNNKQ | 8/8/8/8/4q3/2k4N/5B2/N1K2B2 w - - | A3 | -7 | +41 | -7 | Z u |
| S01 | KRKN | 8/8/8/2k1K3/8/3R4/4n3/8 w - - | [9] #2, ar-Razi (~850) | | | | |
| S05 | KQKRP | 1rk5/8/8/3Q4/8/1p6/1K6/8 w - - | [9] #6 | | | | |
| S03 | KRKBN | k1K5/2n5/8/8/b7/1R6/8/8 w - - | [4] #457, [9] #5, Nunn | | | | |
| S05 | KRKBN | k3b3/n1K5/R7/8/8/8/8 w - - | [9] #5a | | | | |
| S06 | KNNKP | 8/8/1p6/1K6/2N5/3N4/8/k7 w - - | [9] #9 | | | | |
| S07 | KNPKP | 8/8/8/6Pk/4K3/4N2p/8/8 w - - | [9] #7 | | | | |
| S08 | KNPKN | 8/8/8/5KPk/8/8/5N1n b - - | [9] #7a: A2 zug | | | | |
| S09 | KNPKP | 8/8/8/8/4k3/7p/P3K1N1/8 w - - | [9] #8 | | | | |
| S10 | KNPKN | 8/8/8/8/4k3/8/P3K3/5N1n w - - | [9] #8a | | | | |
| S11 | KNPKPP | 8/8/8/8/5p2/4k1p1/4N1P1/5K2 w - - | [39] #5.1, Mandler | | | | |
| S12 | KPPKPP | 8/5pk1/8/2p1PK2/2P5/8/8/8 w - - | [9] #4 | | | | |

Positions S01-S12 are from studies where White wins in an essentially unique way. They all feature a level A zug in both a *try* and the mainline solution and are mainly taken from Beasley [9]. The appendix can accommodate only a few of the solutions so there are plenty of exercises here for the reader.

²⁰ c ≡ castling rights significant, e ≡ e.p. significant, and 5 ≡ 5-1p zug,

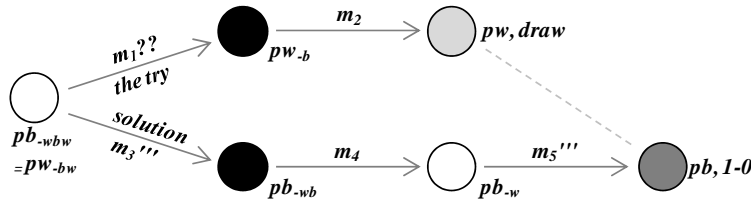


Fig. 2. The Zugzwang Study scenario.

4 Commentary: Statistics, Gems and Studies

Our EGT search has identified a corpus of over 900,000 zugs which may be reviewed in statistical terms, datamined for gems of various sorts, and put in the context of the Chess Study, the chess-engine and the game itself.

In addition to the Level A zug, defined above, two further levels of zug are worth noting. At *Level B*, a zug is merely inconvenient, requiring the winner (loser) to make more (less) moves in some metric before some defined goal. Position P04 and the studies of S01-S12 feature zugs at this level. Note that the number of *moves to goal* may be affected in some metric but not others.²¹ At *Level C*, zugs have no impact on value or depth in any metric but the side to move would rather play a null move than any other: Regan [27] identified P05 as one such, a drawn position in which a null move eases the task of the defender. The likelihood of a win in a theoretically drawn position can be modelled using the concept of a Reference Fallible Player [28-32].

4.1 The Statistics

Pawnless zugs are just 5.26% of the total; pawnful zugs account for the vast majority. There are more pawnful than pawnless positions but the presence of at least one pawn increases the density of zugs by a factor of four. Zugs are also frequent when Knights, parity-bound and unable to *lose the move*²², are present.

Type 3 zugs are clearly rarer than types 1 and 2 and have been the focus to date: it had been established that no pawnless type 3 zug exists with 5 men or less [11].

The presence of an *en passant* feature in a zug has not attracted attention so far, perhaps for three reasons. Type 1-3 zugs in EPZ remain zugs of the same type if the en passant opportunity is removed. Secondly, e.p. constrains two pawns to 14 of the 992 positions available and, at 0.22% of the total as in Table 3, e.p. zugs are few indeed. Finally, the very few type 4-6 zugs had not been discovered.

The next challenges are to trawl for zugs in small EGTs for positions with castling rights, and in large 7-man EGTs. Under the DTZ_{50} metric which recognizes the 50-move Rule, some zugs, q.v. Table 4, lose their status or change type.

²¹ KNNKP position P04: $dtz = dtc = 4m$ if a null move is available; $dtm = 4m$ regardless.

²² Alternating as they do between white and black squares.

4.2 Datamining for Gems

The deepest zugs come closest to not being zugs at all: the shallowest zugs feature the greatest change of advantage achievable by the null move. Relative depths may vary with the depth-metric chosen.

Rare gems have intrinsic value; zugs may be absolutely or type-unique within their endgame and/or have some rare feature. There are only 51 type 2 e.p. zugs, 20 KPPKP and 31 KPPKPP, in which position $p3$ is shallower than position $p1$.

The absence of a pawnless 5-man type 3 zug naturally led to a search for one in 6-man chess. In response to the so-called *Pawnless Trébuchet Challenge* [33], Elkies had conjectured that position Z10, identified three years earlier [8], might just be a pawnless type 3 zug: he did not claim [8] that it was, as incorrectly announced by Roycroft [34]. The search turned to 6-man endgames [12, 33, 35] but the authors confirmed earlier in the work reported here that there were none [29]. Evidence of Elkies' remarkable prescience [36] is that Z10 perhaps comes closest of all 6-man pawnless positions to being a type 3 zug. This is the only position found in which the 2nd player has to avoid the loss by first playing four unique draw-saving moves.²³

With KRBNKNN Z11 derived from KRNKNN Z10 [8], the type 3 challenge became one of reducing the number of Knights in a pawnless zug. The zugs B06 [29, 37], B03 [16] and B02 [38] feature two, one and no Knights respectively though B02 requires *obtrusive*, i.e. obviously-promoted, force.

Surprisingly, there are no type 3 e.p. zugs in 6-man chess but Elkies recalled an 8-man example, N01, from his work [39] on *cold*, i.e. zugzwang, positions. Other than the 393 type 4 KPPKPP zugs, there are just three type 4-6 zugs: one type 4, Z05, no type 5 zugs²⁴ and two type 6 zugs, Z08 and Z09. The type 4 zug is unique in that the value of position $p3$ is worse than the value of position $p1$, but the 1st player is a net winner in type 5 and type 6 zugs. Elkies has provided the first remarkable type 5 zug in Z07 and examples N02-3 of added e.p.-capture rights turning a zug into a non-zug.

Castling rights have not been included in EGTs. However P01 and P02 are the first examples of zugs where castling rights make a difference or not. Elkies [26] has repeatedly provided exemplar zugs, N04-8, where the addition of 0-0 or 0-0-0 castling rights creates a zug, some also featuring significant e.p.-capture opportunities.

4.3 Zugzwangs and Studies

In the Chess Study, White is by convention challenged to draw or win. The appearance of a zugzwang position in a study is notable in itself and, if it is Black to move, suggests that White is just one ply from missing its objective. Mandler's study S11 [40] requires White to revisit a previous physical position 11 plies later but with Black to move: that position is therefore a Level B zug.

A *Zugzwang Study* is defined to be one in which the zugzwang not only appears in the main line of a study in White's favour but also appears as the refutation of a plausible sideline *try* [9]. Fig.2 illustrates the requirements for such a study: a position p

²³ 1. ... Nc5' 2. Nd4 Ne3+' 3. Kd2 Kf2' 4. Ne6 {other moves pressure more} Nxe6'.

²⁴ And MB [31] reports that KPPKPP* (assuming only P=Q allowed) has no type 5 zugs.

must appear in its wtm form pw in the try and in its btm form pb in the main line; White's moves should be essentially unique and Black should play its 'best defence'. Beasley gives some remarkable examples of the genre and his article on the theme, from which most of the study positions S01-S12 are taken, is recommended. The zugzwang study demonstrates that the aesthetic contribution [41] of a zugzwang position to a study must be judged in the context of that study and not in isolation.

5 Summary

The authors have searched the available Nalimov DTM EGTs for 6-man chess to identify all the Level A zugzwangs. Somewhat accidentally, we have discovered three new types of zugzwang to make six types in all: there are no other types.

Work will turn to zugs in the more recently arrived 6-man 'DTC' results [16] which will be compared with those of Nalimov²⁵ and Thompson [12], to Level B and C zugs, and to an examination of the occurrence of zugs in studies [42, 43].

Complementing this review, the full results, including statistics, highlights and lists of all the zugs with their DTM depths, are published on the ICGA website [17]. The zugs may be studied using EGT query services on the web [13, 44] and we look forward to them being mined for gems by the Chess Studies community and others.

Acknowledgments. Our thanks first to Eugene Nalimov for his major contribution of 3- to 6-man DTM EGTs, to Andrew Kadatch for the software used to compress them, and to Marc Bourzutschky, Nelson Hernandez and Kyrill Kryukov for seeding their promulgation. Secondly, our thanks to Marc for his GTBGEN generalisation of Nalimov's TBGEN used to identify the zugs, for his 5-1p and 7-man zugs and for his continued interest. Thanks go to Noam Elkies for rising to the challenge of devising a remarkable string of 'first exemplar' zugs involving e.p., castling or 5-1p chess. Finally, thanks to Ken Regan for a first Level C zug, and to John Beasley, Harold van der Heijden, Soren Riis and John Tamplin for their stimulating dialogue.

References

1. Nalimov, E., Haworth, G.M^cC., Heinz, E.A.: Space-efficient Indexing of Endgame Tables for Chess. ICGA J. 23-3, 148--162 (2000)
2. Kryukov, K.: EGTs Online. <http://kirill-kryukov.com/chess/tablebases-online/> (2006)
3. Roycroft, A.J.: Test Tube Chess: A Comprehensive Introduction to the Chess Endgame Study. Faber and Faber Ltd. (1972)
4. Nunn, J.: Secrets of Pawnless Endings. B.T. Batsford, London. 2nd edition (2004)
5. Nunn, J.: Secrets of Rook Endings. B.T. Batsford, London. 2nd edition. Gambit (1999)
6. Nunn, J.: Secrets of Minor-Piece Endings. B.T. Batsford, London (1995)
7. Beasley, J., Whitworth, T.: Endgame Magic. B.T. Batsford, London (1996)
8. Elkies, N.D.: No. 10965: mutual full-point zugzwang? EG 8-128, 320 (1998)
9. Beasley J.: Creating reciprocal zugzwang studies. EBUR 12-2, 8--12 (2000)

²⁵ First indications [31] are that the 'DTC' and 'DTM' statistics are in full agreement.

10. Wirth, C., Nievergelt, J.: Exhaustive and Heuristic Retrograde Analysis of the KPPKP Endgame. *ICCA J.* 22-2, 67--80 (1999)
11. Haworth, G.M^cC., Karrer, P., Tamplin, J.A., Wirth, C.: 3-5-Man Chess: Maximals and Mzugs. *ICGA J.* 24-4, 225--230 (2001)
12. Tamplin, J., Haworth, G.M^cC.: Ken Thompson's 6-man Tables. *ICGA J.* 24-2, 83--85 (2001)
13. Bleicher, E.: Endgame Service based on Nalimov's EGTs <http://www.k4it.de/index.php?topic=egtb&lang=en> (2009)
14. Tay, A.: A Guide to Endgame Tablebases. <http://www.horizonchess.com/FAQ/Winboard/egtb.html> (2009)
15. Konoval, Y., Bourzutschky, M.S.: Private Communications (2007-8)
16. Bourzutschky, M.S.: Private Communications (2009)
17. ICGA: The ICGA website. Menu: Game-specific information – Western Chess - Endgames. www.icga.org (2009)
18. Thyen, O., Clark, M., Scholze-Stubenrecht, W., Sykes, J.B.: *The Oxford-Duden German Dictionary*. OUP (2005)
19. Hooper, D., Whyld, K.: *The Oxford Companion to Chess*. OUP (1992)
20. Gijssen, G.: An Arbiter's Notebook: Monroi and Other Matters. Chesscafe.com <http://www.chesscafe.com/text/geurt105.pdf> (2006)
21. Gijssen, G.: An Arbiter's Notebook: Interpreting the Rules. Chesscafe.com <http://www.chesscafe.com/text/geurt110.pdf> (2007)
22. Gijssen, G.: An Arbiter's Notebook: Interpreting the Rules. Chesscafe.com <http://www.chesscafe.com/text/geurt129.pdf> (2009)
23. Bourzutschky, M.S.: The 16 "missing" Nalimov files. <http://preview.tinyurl.com/aeqvp2> (2006-07-18)
24. Hernandez, N.: 'Missing 16 received!'. Private Communication (2006-08-08)
25. Bourzutschky, M.S.: Tablebase comparison. <http://www.tinyurl.com/d3wny4> (2006-08-10)
26. Elkies, N.D.: Private Communications (2009)
27. Regan, K.W.: Private Communications (2009-04)
28. Haworth, G.M^cC.: Reference Fallible Endgame Play. *ICGA J.* 26-2, 81--91 (2003)
29. Haworth, G.M^cC.: Chess Endgame News. *ICGA J.* 28-4, 243 (2005)
30. Haworth, G.M^cC.: Gentlemen, Stop Your Engines! *ICGA J.* 30-3, 150--156 (2007)
31. Di Fatta, G., Haworth, G.M^cC., Regan, K.W.: Skill Rating by Bayesian Inference. In: *Proc. IEEE Conf. on Computational Intelligence and Data Mining*, Nashville, USA. (2009)
32. Haworth, G.M^cC., Di Fatta, G., Regan, K.W.: Performance and Prediction: Bayesian Modelling of Fallible Choice in Chess. In: *Proc. Advances in Computer Games 12*. Springer LNCS (2009)
33. Roycroft, A.J.: Announcement of the Pawnless Trébuchet task. *EG* 7-116, 633 (1995)
34. Roycroft, A.J.: Report 1 on the Pawnless Trebuchet task. *EG* 7-117, 645 (1995)
35. Costeff, G.: EG 1-152 online. <http://www.gadycosteff.com/eg/eg.html> (2009)
36. Beasley, J.: Gems Discovered by Computer. *BESN Special Number* 46, 6--7 (2005)
37. Bourzutschky, M.S., Konoval, Y.: 7-man Endgame Databases. *EG* 11-162, 493--510 (2006)
38. Haworth, G.M^cC.: Chess Endgame News. *ICGA J.*, 29-2, 79 (2006)
39. Elkies, N.D.: On Numbers and Endgames: Combinatorial Game Theory in Chess Endgames. In: *Games of No Chance* (ed. R.J. Nowakowski), MSRI 29 (1996)
40. Beasley, J.: *Depth and Beauty: The chess endgame studies of Artur Mandler* (2003)
41. Iqbal, A.: A Discrete Computational Aesthetics Model for a Zero-Sum Perfect Information Game. [http://metalab.uniten.edu.my/~azlan/Misc/phd_thesis_azlan_\(final\).pdf](http://metalab.uniten.edu.my/~azlan/Misc/phd_thesis_azlan_(final).pdf) Ph.D. thesis. University of Malaya, Kuala Lumpur, Malaysia (2008)
42. Costeff, G., Stiller, L.B.: Chess Query Language CQL. <http://rbnn.com/cql/> (2003)
43. Van der Heijden, H.: Endgame Study Database III, 67,691 Studies (2005)
44. Tamplin, J.A.: Multimetric endgame service. <http://chess.jaet.org/endings/> (2009)

Appendix: Some Zugzwang Lines

The 7-man lines are from Bourzutschky [16, 37]. All moves are at least optimal moves given the move-selecting strategy nominated, and beyond that, the key is:

- ''' ≡ only value-saving move (independent of move-choosing strategy),
- '' ≡ the only optimal move, given the strategy nominated, e.g., SC^M, and
- ° ≡ only move.

KPPKP Z04: positions *p1* and *p3* have different depths to mate.

p1, btm: {*dtm* = 25} SM⁻/SM⁺ 1. ... cxd3'' 2. Kxd3''' Kc5'' 3. Kc3''' Kb5'' 4. Kb3''' Ka5 5. Kc4'' Kb6'' 6. Kb4''' Kc6 7. Ka5'' Kb7'' 8. Kb5'' Ka7'' 9. Kc6'' Ka8'' 10. b4 Kb8'' 11. Kb6'' Ka8'' 12. b5 Kb8° 13. Ka6'' Kc7'' 14. b6 Kc8'' 15. Ka7''' 1-0

p2, wtm: 1. K~ Kxd4 = or 1. b~ cxb3''' =

p3, btm: {*dtm* = 15} SM⁻/SM⁺ 1. ... K(c6/d6/e6) 2. Kxc4'' {*dte* = 7} 1-0

KBPKPP Z05: a type 4, draw-win-loss, zug:

p1, btm: 1. ... cxb3''' 2. Bxb3''' =

p2, wtm: SM⁺Z⁺/SM⁻Z⁻ 1. Ke1'' c3'' 2. Bb3 c2'' 3. K~ K(b1/d1/d2) 4. Bxc2'' Kxc2''' 5. Ke1 K(b3/c3) 6. Kd1 Kxb4''' 0-1

p3, btm: SM⁻Z⁻/SM⁺Z⁺ 1. ... Kb1'' 2. Ke3'' Kc1'' 3. Be2 Kc2'' 4. Kd4''' c3'' 5. Bd3+'' Kd2'' 6. Be4 c2'' 7. Bxc2''' Kxc2'' 8. Kc5''' Kb3 9. Kxb5''' Ka3 10. Kc5 Kb3 11. b5'' 1-0

KRBNKNN Z11: a type 3 zug adaption of Z10 which 'just failed' to be type 3.

p1 ≡ *p3*, wtm: 1. N~ N(x)d3#''''; 1. R~ Nxb2#'''' 0-1

p2, btm: {"Black cannot maintain the bind" [8]} 1. ... Nc5 2. Nd4 Ne3+ 3. Kc1 Kg1 4. Bf5 Ng2 5. Nf3+ Kf1' 6. Bh3 Nd3+' 7. Kb1 Nb2 8. Kxb2 {*dte* = 1m, *dtm* = 8m}

KNNNKNN B01: the bK is boxed in but White must avoid a KNNK endgame.

p2, btm: {*dtz* = 17} SZ⁻/SZ⁺ 1... Ndc3 2. Nd7''' Kh7 3. Nf4''' Kg8 4. Nd6''' Kg7 5. Ke5''' Nd1 6. Kf5 Kh6 7. Kf6 Ne3 8. Nf7+''' Kh7° 9. Ng5+''' Kg8 10. Ne4''' Kh7 11. Ne5 Nb6 12. Kf7''' Nbc4 13. Nf6+''' Kh6 14. Nf3''' Nd6+ 15. Kf8 Nef5 16. Ng8+ Kh7° 17. Ng5+ Kh8° 18. Ng6# {10 of White's 17 moves were unique winning moves} 1-0

KRBBKQN B03: a type 3 zug with only one Knight.

p1, wtm: SZ⁺/SZ⁻ 1. Be8 Ka2 2. Bf7 Ka3 3. Bc1+ Kb4 4. Bd2+ Kc4 5. Be8 6. Ne7 0-1

{1. Bxd5 Qxg6+''' {KBBKQ, *dtz* = 62} "is however probably *best defence*" [16]}.

p2, btm: SM⁻Z⁻/SM⁺Z⁺ 1. ... Qh3 2. Ra6+'' Qa3° 3. Rxa3#'' 1-0

KRKN S01: 1.Re3''' Ng1' 2.Kf5' (2.Kf4?? Kd4 pw =) Kd4' 3.Kf4''' pb 1-0

KRKBN S03: 1.Rb6''' Nb5' 2.Ra6+''' Na7+° 3.Kc7''' Be8' 4.Ra3''' (4.Ra2?? Ba4''' pw =) Ba4' 5.Ra2' pb 1-0

KPPKPP S12: 1.Kf4''' (1.Ke4?? Kg6'' pw) Kg6' 2.Ke4''' pb Kg5' 3.e6''' fxe6' 4.Ke5''' Kg4 5.Kxe6''' 1-0